Mathematical Social Sciences 61 (2011) 41–51

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/econbase

A theory of liberal churches

Michael D. Makowsky *

Department of Economics, Towson University 8000 York Rd. Stephens Hall, Room 101M Towson, MD 21250, United States

ARTICLE INFO

Article history:
Received 24 February 2010
Received in revised form
11 October 2010
Accepted 12 October 2010
Available online 21 October 2010

JEL classification:
C63
Z12
D71

Keywords:
Club theory
Moderate religion
Agent-based computational model
Sacrifice and stigma
Heterogeneous agents

ABSTRACT

There is a counterintuitive gap in the club theory of religion. While it elegantly accounts for the success of strict sectarian religious groups in recruiting members and maintaining commitment, it is less satisfactory when attempting to account for groups requiring neither extreme nor zero sacrifice. Moderate groups are always a suboptimal choice for rational, utility maximizing agents within the original representative agent model. The corner solutions of zero and absolute sacrifice, however, are rarely observed empirically compared to the moderate intermediate. In this paper, we extend the original model to operate within an agent-based computational context, with a distribution of heterogeneous agents occupying coordinates in a two dimensional lattice, making repeated decisions over time. Our model offers the possibility of successful moderate groups, including outcomes wherein the population is dominated by moderate groups. The viability of moderate groups is dependent on extending the model to accommodate agent heterogeneity, not just within the population of agents drawn from, but heterogeneity within groups. Moderate sacrifice rates mitigate member free riding and serve as a weak screening device that permits a range of agent types into the group. Within-group heterogeneity allows agents to benefit from the differing comparative advantages of their fellow members.

Said one –"Folk of a surly Tapster tell
And daub his Visage with the Smoke of Hell;
"They talk of some strict Testing of us—Pish!
"He's a Good Fellow, and't will all be well".
– Rubaiyat of Omar Khayyam, translated by Edward Fitzgerald

1. Introduction

Moderate religious groups, at first blush, are easier to understand than are their extreme counterparts. While the peculiar behavior of members volunteering to join groups that require unproductive costs might seem to already stretch the bounds of rationality, certainly membership in relatively low cost groups makes more sense than joining groups with higher costs. Simple economic logic informs us that small costs are preferred to large costs. Secularization hypotheses predicted the eventual demise of religion (Swatos and Christiano, 1999), and under such theories, moderate religious groups, tolerant and lenient, made for a perfect transitional stage from the irrational, costly past to the secular, liberated future. This explanation has lost some of its footing amidst the persistence of religion as a robust social institution. Iannaccone's (1992) theory of utility enhancing sacrifice and stigma requirements (unproductive costs)[1] reconciled much of the dilemma regarding the attractiveness of high cost groups. Unproductive costs, he showed, can serve to incentivize members to dedicate a greater share of their productive resources to the group and screen out potential free riders. This impressive piece of rational choice theorizing reconciles the behavior of the most devout, those willingly sacrificing so much of their potential productive capacity. Counterintuitively, perhaps even ironically, the model is considerably less adept at explaining the empirical reality of successful moderate religious groups. It is the members of low cost groups that fail to conform to the rational predictions of the club theory of religion. While the devout and the devoutly secular emerge as viable outcomes, the scores of moderately dedicated, the Unitarians, ecumenical, "mainstream" Protestants, and twice-a-year Catholics, to name a few, would appear mired in the suboptimal. We can push this concept farther. The original model would indicate that only religious groups operating at the utility maximizing level of sacrifice and stigma would be extremist groups whose members have little or no contact with the outside world. Relatively more sectarian groups such as the Church of Latter Day Saints (Mormons)

* Tel.: +1 703 608 1366.
E-mail addresses: mikemakowsky@gmail.com, mmakowsky@towson.edu.

0165-4896/$ – see front matter
doi:10.1016/j.mathsocsci.2010.10.001

[1] Sacrifice and stigma can be broadly (and more formally) defined as destroyed resources and forgone extra-group (secular) utility-producing opportunities. The "sacrifice" mechanism employed in our formal model falls best under the rubric of prohibitions, specifically prohibited secular productive opportunities.

and Jehovah's Witnesses could also be considered to be mired in the suboptimal intermediate. Within the structure of the original model agents who would prefer any amount of religious sacrifice would in fact prefer the group demand complete prohibition of all things secular, perhaps even all things outside the bounds of the congregation.[2]

In this paper we present a theoretical extension of the original Iannaccone sacrifice and stigma model that generates a distribution of religious groups across a range of potential sacrifice requirements, reconciling the model not just with the feasibility of moderate religion, but with the possibility of a variety of disparate groups coexisting within a population. The viability of moderate sacrifice imposing groups is dependent on extending the model to accommodate agent heterogeneity, not just within the population of agents drawn from, but heterogeneity *within groups*. Moderate sacrifice allows groups to offer a utility optimizing mixture of screening and incentive adjustment that reduces agent free-riding,[3] while at the same time allowing agents to benefit from the differing comparative advantages of fellow members.

Free-riding and the representative agent

While secularization theories come in a variety of forms, economic theory is more inclined to point to strategic behavior by rational members as the most serious impediment to the successful provision of religious club goods. Under the auspices of the prisoner's dilemma, and the free-riding it should entail, groups and factions beyond a minimal size should fail to corral the efforts of their members (Olson, 1965). Various theories of the firm and other forms of collective action go to great lengths to demonstrate how such problems are overcome (Ostrom, 2000; Williamson, 2002). Religious groups typically operate without any wage or contract structure, and still produce a club good largely dependent on members whose efforts are, at best, difficult to monitor. Further, these groups often impose non-trivial costs on their members. At first glance, this imposition of costs would appear to present an exception to the first law of demand. Iannaccone's (1992) theory of sacrifice and stigma reconciles this apparent tension between theory and reality, positing that these costs, in the form of forsaken extra-group (secular) opportunities, make it possible for groups to overcome problems of free-riding on behalf its members. The quality of the good changes with increasing unproductive costs, causing a shift in demand that, observed in a static/homogeneous good context, might give the impression of an upward sloping demand curve. Rather, greater sacrifice requirements are increasing the quality of the club by serving as a clever screening mechanism and aligning member incentives with group preferences. Acceptance and application of this theory has become increasingly numerous in the scientific study of religion, within both sociology and economics (Berman, 2000; Stark and Finke, 2000; Berman, 2003; Keister, 2003; Cosgel and Minkler, 2004; McBride, 2008; Berman, 2009).

The sacrifice and stigma theory is built using a representative agent, with groups comprised of agents homogenous across all attributes, most notably wages. The theory predicts that agent utility may be increasing with the imposition of unproductive costs. In the original model, utility is monotonically increasing in both directions from a global minimum over the range of unproductive prohibitions imposed on non-group (secular) productivity. As a result, within the original theory a group will maximize the utility of its members either by enforcing complete sacrifice of all secular activities or no sacrifice at all. Applying this construct to different types of representative agents, as well as varying any and all of the model parameters, leads to differing conclusions as to which corner solution is optimal, but never to the possibility of a utility maximizing amount of sacrifice within the intermediate values.

As with most simplifying assumptions, the representative agent assumption is empirically false and incredibly useful. The model's outcome that religious groups can only successfully avoid crippling free-riding by demanding an absolute sacrifice of all things secular is, of course, false as well. In extending and reconstructing the Iannaccone model of religion to allow for agent heterogeneity, both within groups formed and the greater population agents are drawn from, we find that intra-group wage differentials are the key to allowing for the viability and dominance of moderate religious groups within the model. This emergence is a direct result of the ability of members to free-ride off the greater efforts of members of the congregation with relatively lower opportunity costs of time. Small sacrifice requirements serve as barriers to entry for higher wage agents. Members find it utility maximizing to stay within a particular group due to their capacity to free-ride off of fellow members and their unwillingness to defect to a group that demands greater sacrifice. These moderate costs, varied and staggered across a religious landscape, serve to segregate the modeled population's religious groups by wages and preferences. Thus the sacrifice mechanisms works both to realign incentives within the group, as demonstrated by Iannaccone (1992), and as a screening device for prospective members with unobservable attributes, as used by Berman (2000, 2003) and Iannaccone (1994).

2. Survey data

Religious groups in the United States come in a tremendous variety, but there are few that would be considered "extreme" in their demands of members, on both an absolute scale and relative to the distribution of religious groups. Instead, we find a spectrum largely dominated by groups requiring contributions of wealth, time, and energy rarely more than 10% of an individual's "full" income. We define full income in the Beckerian sense as the individual's maximum productive capacity (Becker, 1965).

The 2005 Baylor Religious Survey presents considerable evidence of the vitality and dominance of moderate religious groups in the United States. While it does not rule out the existence of extremist groups, it does demonstrate their limited profile. Using results from questions regarding time spent attending religious services (mass), volunteering for their congregation, and engaged in religious service activities, we create an approximation of time spent dedicated to religious groups during the last year. We use responses to questions regarding income and hours worked the previous week to impute a wage rate. This imputed wage rate is translated into a respondent's potential "full income" (Becker, 1965). From this we calculated the fraction of a respondent's estimated full income that was dedicated to religious activity associated directly with his or her congregation, R_{FRAC} (see Appendix for a breakdown of survey questions used and the imputation of values). Summary statistics of the survey variables used as inputs into the calculation of R_{FRAC} and a description of the response format in the Baylor survey are included in Table 1. In the Baylor survey data, the mean respondent R_{FRAC} was 3.56% with a standard deviation of 5.37% (see Table 1). The interquartile range of R_{FRAC} is 0.05%–5.2%, offering further evidence that the majority of respondents were members of religious groups that demanded commitments that we casually classify as "moderate".

2 Liebman (1983) anticipated this conundrum of the moderate a decade earlier, positing a sociological theory of religion as naturally extremist. He took the view that religion, operating in a simplified social vacuum, would be naturally expansive, seeking to overtake the social sphere that encompassed believers, and prohibit all that could not be incorporated within it. This was to him, the easy part. It was moderate religion that he felt was left begging an explanation. Why would religion emerge as an institution that sought to limit itself? The conclusions of the original Iannaccone model of sacrifice and stigma begs a similar question.

3 The more correct term would be "easy-riding" (Cornes and Sandler, 1986) but we use the more general "free-riding" throughout.

Table 1
2005 Baylor religious survey.

	Mean (Std. dev.)	Survey scale (# of response options)	Conversion
Income	66,452 (52,815)	Range [7]	Integer
Hours worked last week	26.34 (21.70)	Integer	No
Full income[a]	233,781 (437,911)	N/A	N/A
Tithe per year (dollars)	1385.24 (2609.93)	Range [12]	Integer
Tithe as% of income	2.61% (5.06%)	N/A	N/A
Hours volunteering for religious group per year	17.65 (38.15)	Range [5]	Integer
Hours is religious activities per year (10 items)	129.43 (228.22)	Range [4]	Integer
Hours at religious service/mass per year	27.85 (31.41)	Range [9]	Integer
Religious fraction of full income[b]	3.56% (5.37%)	N/A	N/A

Standard deviations in parentheses.
[a] Full Income is calculated by extrapolating hours worked last year to calculate an hourly wage. This wage is then applied to a 16 h work day, and multiplied by 365.
[b] Religious Fraction of Full Income is the wage value of time spent in religious services, volunteering to the congregation, and religious activities plus tithing, as a fraction of full income. See Appendix for data imputations.

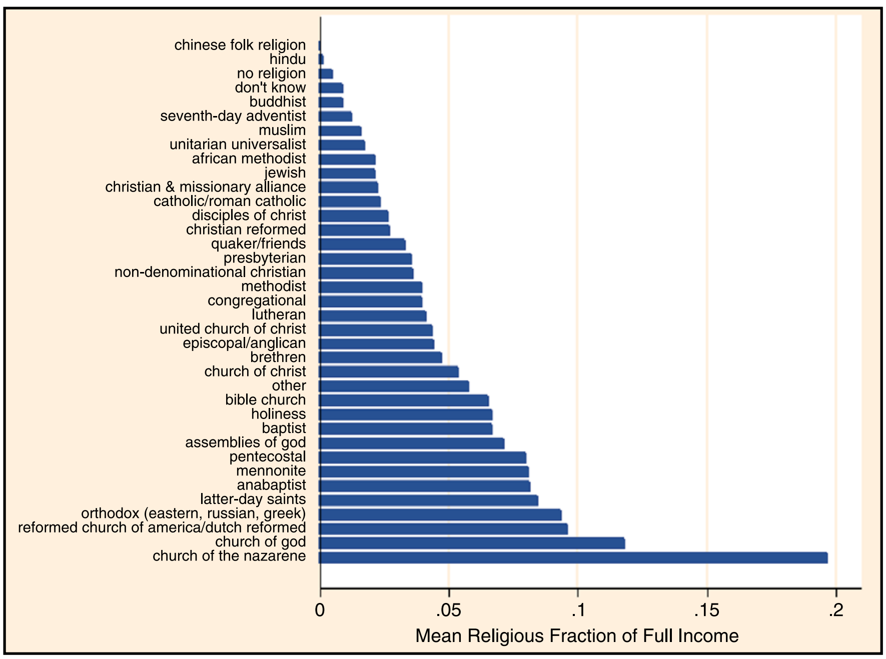

Fig. 1. Mean Fraction of full income dedicated to the respondent's religious congregation, organized by denomination. Denominations with an insufficient number of wage and/or hours worked responses are excluded (see Fig. 2 for commitment in hours for additional denominations). See Appendix for data imputations.

While no explicitly quantitative measure of "sacrifice and stigma" is possible, there has been a tremendous effort to categorize American religious denominations with regards to their level of "strictness", "sectarian-ness", "tension", or sacrifice (Johnson, 1963; Stark and Bainbridge, 1980; Iannaccone, 1997b; Steensland et al., 2000). In Figs. 1 and 2 we can see that the fractions of full income and hours dedicated to congregational activity per year within the different respondent denominations corresponds nicely with the level of sectarian-ness generally associated with those groups.

At the upper end of the R_{FRAC} spectrum in Fig. 1, we see find Pentecostals, Church of Latter Day Saints, Mennonites, and Church of Nazarene with mean responses correlating to 8% or greater fractions of full income committed to their congregations, with Church of the Nazarene identifiers topping the list, approaching 20%. Each of these groups is considered as strict and, compared to most groups with lower mean values, more sectarian (Hoge, 1979; Iannaccone, 1994; Woodberry and Smith, 1998). Closer to the middle of the pack we find self-identified Methodists, Congregationalists, and Lutherans, groups which most scholars would comfortably identify as moderate (Smith, 1990; Steensland et al., 2000), all hovering around a mean R_{FRAC} of 5%. At the lower end of the spectrum we find Unitarian Universalists, Jews (broadly categorized), and Catholics.[4]

The broader point to be taken from Figs. 1 and 2 is the prominence of groups commonly associated with the American religious mainstream and the relatively moderate demands they place on their members. The American religious groups represented in the sample of survey respondents allow members to retain the bulk of their resources, and remain highly productive outside of their groups. At the same time, 4% of full income is a non-trivial fraction of an individual's productive capacity.

Moderate groups would appear to be not only viable in the United States, but in fact the dominant strand in the religious mainstream.

3. The agent-based computational model

The sacrifice and stigma model, as constructed in the literature to date, is unable to generate moderate religious groups without

[4] We also find at the lower end of the spectrum a handful of religious groups, some not generally associated with moderate requirements, that do not fit the mold of traditional western congregational religion, such as Buddhism and Islam. The commitment of these individuals is likely not properly represented by our metrics.

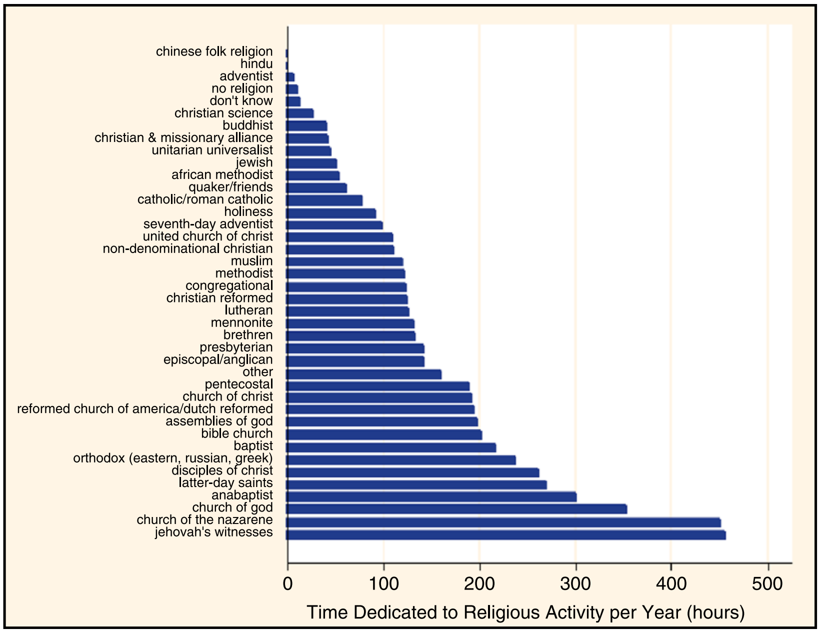

Fig. 2. Mean hours dedicated to the respondents congregation, by denomination. See Appendix for data imputations.

severely limiting its primary mechanism for mitigating free-riding. Further, the general model relies on a Nash-Equilibrium assumption of identical behavior amongst representative agents and is not able to accommodate in-group heterogeneity and retain analytical tractability.[5] As such, it is useful to extend the model into an agent-based framework to accommodate in-group heterogeneity amidst strong population heterogeneity.

We construct our computational model with mathematical underpinnings explicitly based on Iannaccone's original model. Adapting the original model to accommodate a heterogeneous, agent-based framework allows us to test the implications of the club model of religion for the different types of groups vying for members in a religious marketplace. In this paper we specifically explore the viability of moderate sacrifice groups, their ability to recruit members, and the level of commitment that moderate sacrifice engenders amongst its members. Given a distribution of agents with heterogeneous wages, we will also investigate the distributions of wages within groups and average wages across groups. Similar to the Iannaccone model, we employ a reduction in the private productivity of members as the "sacrifice" mechanism, as opposed to the destruction of resources or limitations on interaction with other agents. This mechanism falls best under the rubric of "costly prohibitions". For simplicity, we will simply refer to these costly prohibitions as sacrifice.

The club model of religion begins with the premise that agents internally produce their own utility. This production relies on two inputs which are similarly produced by the individual in their secular (private) endeavors and their religious (group) endeavors. Both secular and religious production require commitments of time and money, both of which are limited in supply. Time endowments are homogenous across individuals, while money is a function of wages that are heterogeneous and exogenously assigned across the population. What makes the production of the religious input unique is the interdependence of religious production with other members of the group. This interdependence invites members to free-ride—to be a member of the group and benefit from the religious production of other members while in turn neglecting her own religious production. Iannaccone's crucial insight was that the imposition of costly sacrifice and stigma requirements could mitigate the free-rider problem, resulting in rational members whose choice to engage in more religious production increased not just their own utility, but the utility of all other members.

Individuals are heterogeneous in their wages, but identical in their basic preferences. Similarly, religious groups are heterogeneous in their sacrifice and stigma requirements, but are identical in their capacity to produce the religious "club good". What can, in turn, emerge is a religious economy within which some groups succeed in attracting members and others fail. Within this economy, individuals will decide how best to invest their scarce resources—whether to produce their own utility by allocating their time and money to secular endeavors or to their chosen religious group. A spectrum of agent choices will also emerge, including the secular independent, the devout group member, and everything in between.

In the model constructed, each agent produces her own utility with a constant elasticity of substitution (CES) production function, with inputs of a secular, private good S, and a religious, club good K, preference parameters d_S and d_K, and a substitution parameter β. S and K are classic "Z-good" arguments in the utility function (Stigler and Becker, 1977). K is produced with a Cobb–Douglas production function with constant returns to scale and inputs R_i, the individual's contribution, and Q_g, the "quality" of the other group members' contributions, with output elasticity parameters α and $1-\alpha$

$$U_i = (d_S S_i^{\beta} + d_K K_i^{\beta})^{1/\beta}$$
$$K_i = (R_i^{\alpha} Q_g^{1-\alpha}). \tag{1}$$

The group quality input, $Q_{i,g}$ is defined as a function of the average input R across agent i's neighbors ($j \neq i$), a scalar $s > 0$,

[5] Utility is U-shaped over the range of sacrifice rates. Changing the parameterization (or functional form) can reduce utility to being linear and decreasing over sacrifice, but the shape never flips (there is never an intermediate global maximum). Berman's (2000) formal model of sacrifice allows for utility maximizing intermediate requirements, but this is only possible by recasting sacrifice in manner such that it only serves as a screening device, and does not create a substitution effect incentivizing agents to reallocate resources from private production to group production. This simplifying of the model was necessary for Berman's analysis of subsidies, but reduces broader applicability to free-rider problems in groups for tractability in a specific problem. Within the paper Berman discusses a more general model of prohibitions, but this is left out of the formal model for reasons of tractability.

and the number of agent i's neighbors, n_g, that are members of the group, g, being evaluated

$$Q_{i,g} = \widetilde{R}_{g,j\neq i} \cdot s\left(1 - \frac{1}{1+n_g}\right). \quad (2)$$

$Q_{i,g}$ is strictly increasing in n_g, with diminishing marginal returns ($Q' > 0, Q'' < 0$). This formulation of $Q_{i,g}$ is an important mathematical change from the original model. The original model hinges on a Nash-Equilibrium assumption ($R_i = R_{j\neq i}$) in order to maintain analytical tractability. In order to accommodate deeper (population and in-group) heterogeneity, the Nash-Equilibrium assumption must be abandoned. As such, $R_i = R_{j\neq i}$ no longer necessarily holds true and the model ceases to have a closed-form equilibrium solution.[6] Instead, in our model agents are able to observe local agent behavior different from their own, and in turn inform their own decision-making. Operating in a computer-aided framework, however, we are less dependent on finding closed-form solutions.[7] The utility function, for any given value of $Q_{i,g}$, contains only a single, global maximum, which allows the luxury of employing the relatively simple golden mean search optimization algorithm (Press, 2002).

S and R are both Cobb–Douglas produced with inputs of goods, x_S and x_R (prices p_S and p_R); and time v_S and v_R; input elasticity parameters a and b; and production capacity parameters A_S and A_R. A_S is the dimension in which group sacrifice is implemented[8]

$$\begin{aligned} S_i &= A_S(x_{i,S}^a v_{i,S}^{1-a}) \\ R_i &= A_R(x_{i,R}^b v_{i,R}^{1-b}). \end{aligned} \quad (3)$$

Agents are exogenously endowed with a heterogeneous wage rate, w_i, and a uniform time endowment $V = v_{i,S} + v_{i,R}$. Using the envelope theorem, we can construct shadow prices πR and πS.[9] With agent-specific shadow prices established, agent choice is an exercise in standard optimization constrained by the agents' exogenously endowed full income $(p_S x_{i,S} + p_R x_{i,R}) + (w_i v_{i,S} + w_i v_{i,R}) \leq I_i$ (Becker, 1965), defined as the value of goods purchased and wages forgone to time invested, where w is the agent's wage rate and p_S and p_R are the prices for secular (x_S) and religious goods (x_R).

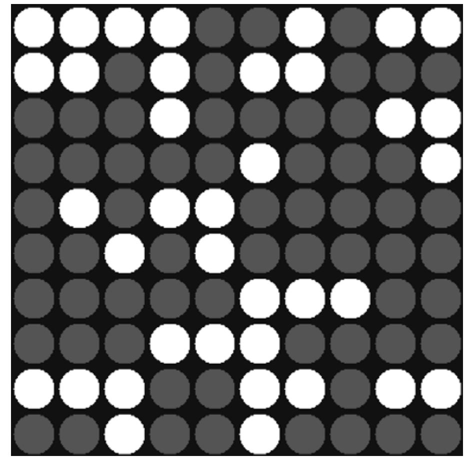

Fig. 3. 10 by 10 lattice (100 by 100 lattice used in actual simulations). Different colors correspond to group membership.

In evaluating $Q_{g,i}$, agent i is evaluating agents currently occupying patches in her neighborhood who are members of group $g \in G$ where $G = \{0, 1, 2, \ldots, m\}$. The groups in set G are differentiated along required member sacrifice in private productivity parameter A_S^g, where:

$$A_{g,S} \begin{cases} 1 - 0.9^{(m-g)} + \varepsilon & \text{if} \quad g > 0 \\ 1 & \phantom{\text{if}} \quad g = 0. \end{cases} \quad (4)$$

The sacrifice that a group enforces comes at the expense of $A_{g,S}$, where the first group ($g = 0$) offers member private productivity parameter $A_{0,S} = 1$ (no sacrifice) and the final group requires $A_{G-1,S} = \varepsilon$ (complete sacrifice, where ε is defined as an arbitrarily small value to prevent division by zero). The resultant sacrifice is $1 - A_{g,S}$.[10]

The computational model exists as a two dimensional lattice (Fig. 3) not unlike a checkerboard, on which agents occupy spaces identified as "patches". Agents are one to a patch, and have a set of eight neighboring patches (four adjacent and four on the diagonals) whose occupants make up their "neighborhood". Within this spatial context agents engage in local optimization, choosing the group and personal investment in club production that maximizes utility in their own unique local context. Given that each agent holds a unique set of coordinates and neighbors during any time step of the model, the spatial construct is an important source of agent heterogeneity in the model.

3.1. Model steps

A run of the model consists of initialization followed by a set number of time steps, summarized by the following time structure:

Step [$t = 0$] *initialization*. The model creates and places agents randomly, one per patch. Agents are heterogeneous across wages, pulling random values from a lognormal wage distribution. Agents are randomly assigned an initial group from a set of $m+1$ different groups. Upon their creation, agents optimize their values of R and S as a function of their wage and the sacrifice required by their

6 The computational model generates outcomes equivalent to the Nash Equilibrium outcome of Iannaccone's original model when constrained to a representative agent. The implied two-group outcome possibility can also be generated if two agent types are employed.

7 The model is written in Java 1.5.1 using the MASON agent modeling library (Luke et al., 2005).

8 Both the original and agent-based model results are sensitive to model parameterization and, in turn, functional form. Substitutability, β, must be greater than the output elasticity of R, α, for the sacrifice mechanism to work (Iannaccone, 1992). If β is too low, and club goods and private goods are sufficiently complementary, the resulting interdependence renders free-riding a suboptimal strategy and sacrifice and stigma unnecessary. Additionally, the output elasticities for goods and time within the production functions for S and R must be different, with R weighted towards the time input relative to S ($a > b$). The necessary differentiation of weightings ($a \neq b$) is intuitive: if the two goods are indistinguishable, then the lower priced one will always be favored, and any sacrifice will cause the club to fail. If $S()$ and $R()$ employ inputs of goods and time differently, then the ratio of shadow prices depends on the agent's full income and her opportunity cost of time. It is assumed that club production places greater emphasis on its members' time than does private production (i.e. $a > b$). These limitations on β restrict the range of functional forms of utility production (e.g. groups imposing any unproductive costs will always be suboptimal under Leontief and Cobb–Douglas produced utility). The sacrifice mechanism only works within functional forms where there can be significant substitution of club goods for private goods, but limited substitution of material goods for time in club good production.

9

$$\begin{aligned} \pi_S &= p_S\left(\partial x_S^*/\partial S\right) + w_i\left(\partial v_S^*/\partial S\right) \\ &= 1/A_S\left(p_S\left(aw_i/(1-a)p_S\right)^{1-a} + w_i\left(aw_i/(1-a)p_S\right)^{-a}\right) \\ \pi_R &= p_R\left(\partial x_R^*/\partial S\right) + w_i\left(\partial v_R^*/\partial S\right) \\ &= 1/A_R\left(p_R\left(bw_i/(1-b)p_R\right)^{1-b} + w_i\left(bw_i/(1-b)p_R\right)^{-b}\right). \end{aligned}$$

10 Different bases were tested for the sacrifice function. As the number of groups is increased, the model becomes more fine-grained, but at the cost of speed and ease of data collection and analysis. The formula employed allows for finer grained analysis at the lower end of the sacrifice spectrum and sufficient variety at the higher end, while limiting the model to a tractable number of groups to simulate.

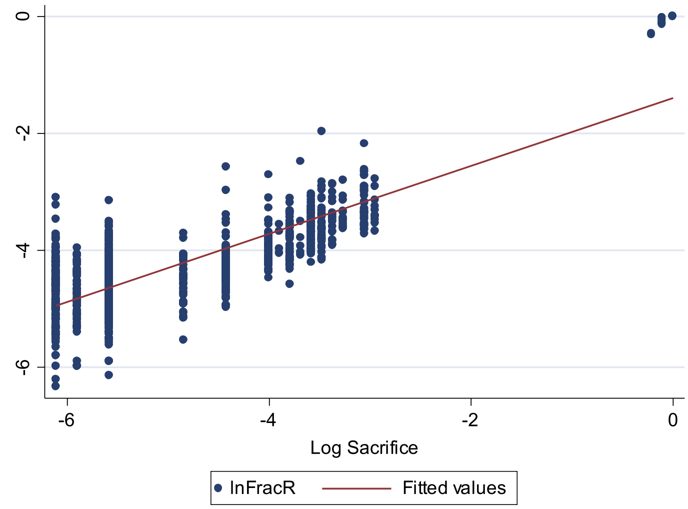

Fig. 4. Log R as fraction of full income over log sacrifice.

initial group in an iterated Cournot–Nash solution to a game that the agent plays against herself. This is the only time that agents will act without knowledge of their neighbors.

Step $[t > 0]$. Each model step consists of shuffling the order of agents and progressing through their ranks one at a time. When it is agent i's turn, she will evaluate $Q_{i,g}$ for each prospective group, g, that is represented in her neighborhood.[11] The optimal R and S are determined for each potential group, with the agent joining the utility maximizing group. The choice of group for an agent is a function of her wage, each group's respective sacrifice demanded, the quantity and commitment (in terms of R) of the representatives of each group in her neighborhood, which is in turn a function of their wages and the decisions made by their neighbors, and so forth. The patterns of groups that emerge within and across runs of the model display a richness that is, perhaps, surprising for a model rooted in a standard, two input, CES structure.

4. Experiment and results

The model was set to run with a 100 by 100 lattice, filled with 10,000 agents, divided amongst 60 initial groups. The income distribution is lognormal with a shape parameter of one. All preference parameters are set to unity. The scalar multiplier, s, in the calculation of Q is set to 1.25.[12] Parameter assumptions are summarized in Table 2.

The β and mean wage, μ, are the key parameters tested. The experiment varies the β in increments of 0.1 between 0.4 and 0.9.[13] The mean wage is varied in units of 0.5 between 0.5 and 4.0. Each combination of β and μ is run 50 times, with 100 turns constituting a run. In the course of a turn, each of the 1089 agents is activated in random order. When an agent is activated, she surveys her local neighborhood and chooses the utility maximizing group.

Table 2
Model parameters.

Parameter	Related function	Value
d_s, d_k	$U_i = (d_S S_i^\beta + d_K K_i^\beta)^{1/\beta}$	1
s	$Q = \widetilde{R}_{j\neq i} \cdot s\left(1 - \frac{1}{1+n}\right)$	1.25
α	$K_i = (R^\alpha Q^{1-\alpha})$	0.3
a	$S = A_S(x_S^a t_S^{1-a})$	0.7
b	$R = A_R(x_R^b t_R^{1-b})$	0.3
G	(Number of groups)	60
p_S, p_R	(Prices of goods)	1

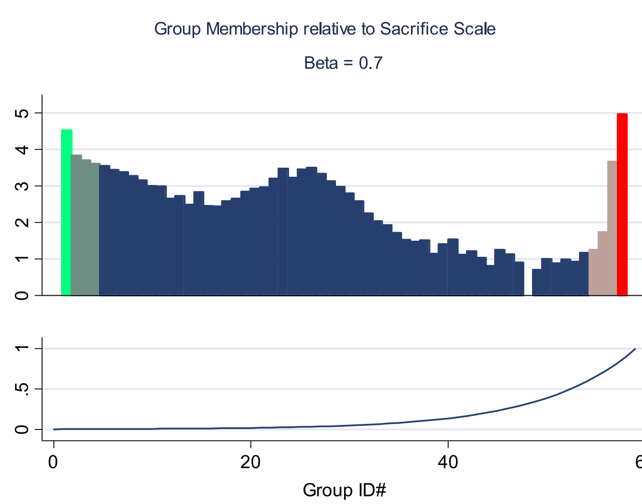

Fig. 5. Distribution of members across groups, sacrifice scale by group.

The sacrifice and stigma theory predicts that groups require unproductive costs as a means to solving the free-riding problem. It is to be expected that as unproductive costs increase, so does the commitment of resources to the club by members. This is exactly what happens in the simulation model, as seen in Fig. 4, with the fraction of full income dedicated by agents plotted against the level of sacrifice they required by the group each agent chooses to join.

The sacrifice profile of a population of agents is represented in Fig. 5 as a bar chart of membership size across groups. Each bar represents $\log(n_g)$ of a different group, in increasing order of sacrifice required by the group $g \in [0, 59]$. The actual sacrifice level of the group is between 0% and 100%, shown in the lower portion

[11] The set of groups considered always includes the zero-sacrifice group, regardless of whether a member resides in the agent's neighborhood.

[12] The scale parameter, with other parameters held constant, shifts the population profile towards the club good, and in terms of sacrifice, towards the more extreme. As s increases, the size of K relative to S increases for all values of Q. The rewards to sacrifice are, in turn, increasing with s. This linearly impacts the commitment profile of the population, but does not meaningfully impact the results of the model discussed here when constrained to a reasonable range of values. Future work related to the success of "mega churches" may wish to explore the impact of the scale parameter in other contexts.

[13] As Iannaccone notes in the original article, the sacrifice mechanism only succeeds can only be successful if $\beta > \alpha$.

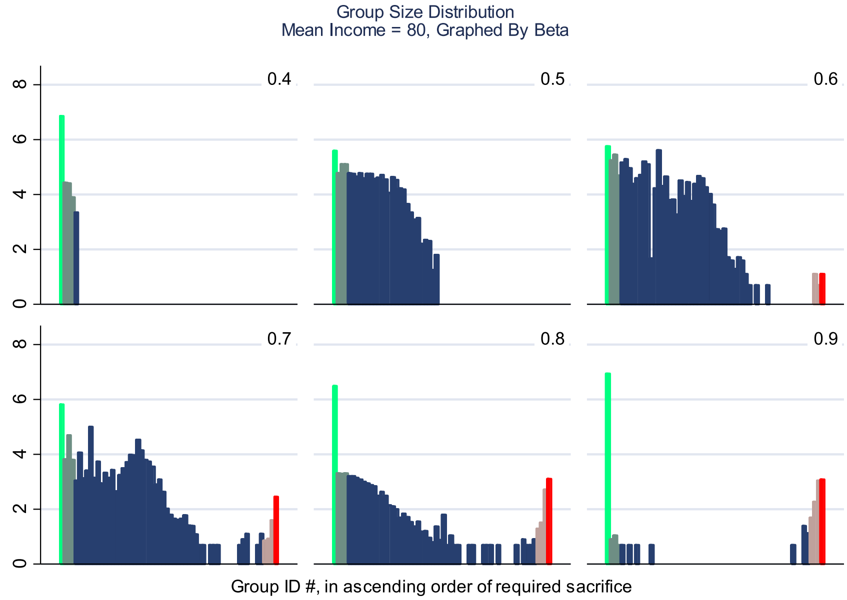

Fig. 6. Distribution across beta—high income by group.

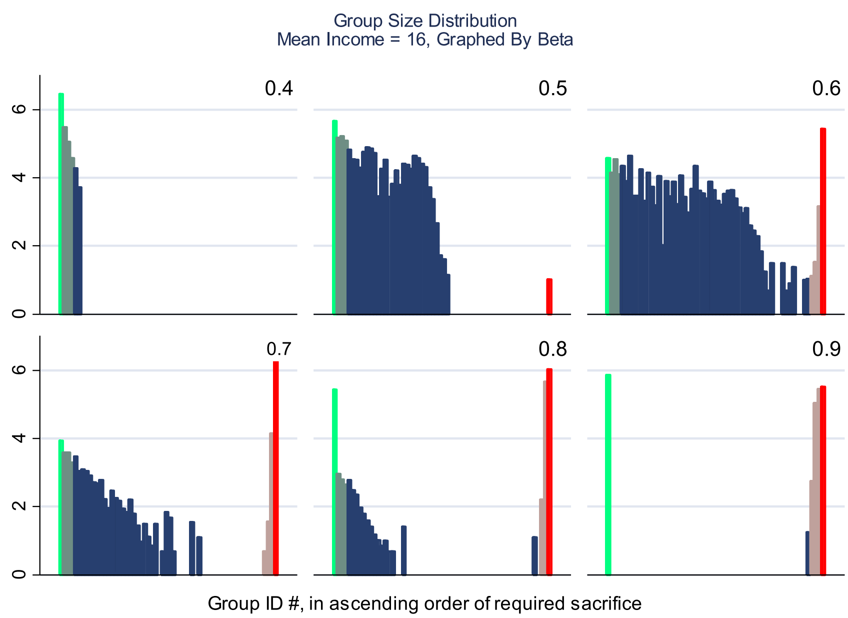

Fig. 7. Low income population, distribution across beta, by group.

of Fig. 5, increasing in accord with Eq. (4). Fig. 5 shows the overlapping results of a series of runs with identical beta parameterization, $\beta = 0.7$, but a range of mean population full incomes, $\mu \cdot T = [8, 64]$ in 8 unit increments. The entire range of sacrifice levels allow for groups that are potentially viable in the population. It is worth noting that we only simulate relatively continuous distributions of wages, hence the generally uninterrupted distribution of viable groups. Discontinuities, or bunching, within the range of wages in the population would result in corresponding gaps in the range of sacrifice rates associated with active, viable groups.

Figs. 6 and 7 allow a comparison of different mean population incomes, again noting the breadth of sacrifice levels that make for viable groups. In the relatively high population income simulations (Fig. 6), moderate religion is robust across all substitutability parameter values (β). The appeal of liberal, low sacrifice, groups increases as β decreases, especially for liberal groups whose sacrifice requirements are greater than zero. In simulations with lower average population incomes, however, moderate religion remains viable, but only so long as substitutability remains sufficiently weak (Fig. 7). When $\beta = 0.9$, meaning private and group activities are nearly perfect substitutes for one another, we see results analogous to what would be derivative of the original Iannaccone model. Specifically, the only groups that are successful are those demanding either very large sacrifice or none at all. Regardless of income, greater substitutability polarizes the population, pushing them towards the highest and lowest sacrifice groups. Weak substitutability is sufficient, and nearly necessary for moderate sacrifice groups to remain viable over time within a population of heterogeneous agents. Relatively wealthier income distributions favor moderation, but high income is neither necessary nor sufficient for the viability of moderate sacrifice groups.

5. How moderate groups persist

In Iannaccone's original model, utility monotonically increases in both directions from the global minimum. Only the corner solutions hold the potential for optimality, leaving the agent with the choice of either sacrificing all of her secular productive capacity or none of it. In our agent-based model, however, we find that a continuum of groups is possible. While the continuum from zero to 100% sacrifice may have significant discontinuities, depending on the size of the population simulated and the parameters governing the model, the viability of groups demanding intermediate levels of sacrifice is evident. How is this possible given the underlying mathematics governing the model?

The key is the heterogeneity of wages across agents. Agents who earn lower wages place a greater fraction of their productive capacity into time-intensive religious endeavors from which other group members benefit.[14] Population wages are pulled from a continuous distribution and each group will be composed of members with different, though perhaps similar, wages. When group members earn a variety of wage rates, an individual agent has incentive to find a group whose members are *relatively poorer* than she is. In the classic free-rider's gambit, she wants fellow members who put more into the group than she does.

There are limits to this incentive, however. Dependent on the parameterization of the output elasticities of time and goods to the production of S and R, while lower wage agents have a *comparative* advantage (Eq. (5)) in the production of religious group goods, higher wage agents have an *absolute* advantage (Eq. (6)) in the production of all goods. In Eqs. (5) and (6) R^*, S^* represent the optimal choices of R and S, and $R^{\max}$ is the maximum value of R possible.

$$\text{if } b < a \text{ and } w_i > w_j \text{ then } \frac{R_i^*}{R_i^* + S_i^*} < \frac{R_j^*}{R_j^* + S_j^*} \tag{5}$$

$$\text{if } b > 0 \text{ and } w_i > w_j \text{ then } R_i^{\max} > R_j^{\max}. \tag{6}$$

Agents will only desire relatively poorer fellow members so long as their comparative advantage outweighs their absolute disadvantage such that $R_j^* \geq R_i^*$.

The potential for utility enhancing moderate sacrifice can be demonstrated through a numerical exercise. This exercise involves a two-player game of differing wages, informed by various results from the simulations of our agent-based model, and can illustrate how the model can allow for utility maximizing moderate sacrifice. Moderate sacrifice remains viable so long as the returns to increased sacrifice are positive, specifically so long as $\frac{dU_i}{dR_j}\frac{dR_j}{d\pi_S}^{(+)} + \frac{dU_i}{dS_i}\frac{dS_i}{d\pi_S}^{(-)} - \frac{dU_i}{dR_i}\frac{dR_i}{d\pi_S}^{(+)} > 0$.

We here recast the original model in terms of a group with two members, agents i and j.

$$U_i = (b_S S_i^{\beta} + b_K K_i^{\beta})^{1/\beta} \tag{7}$$

$$K_i = (R_i^{\alpha} R_j^{1-\alpha}). \tag{8}$$

Q_i has here been replaced with R_j, the religious group production of the sole other member. Shadow prices are calculated as before (see footnote 3). In the numerical example of Iannaccone's original paper, Marshallian demands are derived with standard optimization and a Nash Equilibrium assumption,[15] which we employ here.

Table 3
Log-linear regression of maximum wages in moderate groups.[a]

	Group maximum wage
Log μ	1.991 (0.012)
Log A_g	145.143 (2.629)
R^2	0.9325

[a] $N = 3751$. The regression is run over the subset of "moderate" groups: $g \leq 20$; $\beta = 0.7$.

Agent wages within our earlier simulation model are pulled from a lognormal distribution. Running the model for a 100 time steps allows agents to find their respective optimal groups. As expected, higher wage agents are attracted to lower sacrifice groups, and vice versa. This sorting process, however, results in groups whose within-group median and maximum wages are more heavily skewed than population wages, demonstrating possible power law characteristics likely derivative, at least in part, from the exogenously set scaling of sacrifice across groups (see Fig. 8).

We factor this skewing of group maximum wages into the two-player game by making agent j's income a function of agent i's income and the sacrifice level of their two-player group, such that $w_j = w_i^{\phi} A^{\theta}$. We chose the parameters of this function by returning to the simulation data generating in the previous experiments.[16] Agent i's preferred outcome is to find a group with the lowest sacrifice level *within which he has the highest wage*. Agent i, who we can think of as an "average" agent,wants to give up as little as possible, but have everyone else contribute more to the group than she does. Our "average" agent, i, wants to predict what the highest wage (other than her) in each group will be, and join a group where she can be the highest wage agent. Using the simulation data, we can regress the highest wage of members of each group. $\max(w_i)g$, on the productivity factor of the group, A_g (where $1-A_g$ is the sacrifice requirement), and the mean wage of the entire population, μ

$$\max(w_i)_g = \beta_1\mu + \beta_2 A_g + \varepsilon_g. \tag{9}$$

What we expect is that the maximum wage of the group will be increasing with both the productivity parameter (i.e. low sacrifice groups will have higher maximum wages) and the mean wage of the population, and that is exactly what we find.

We use these results, seen in Table 3, to parameterize the numerical exercise: $w_j = w_i^2 A^{145}$.[17] For this exercise, we reduce the rest of the group to a single agent, j, who represents the maximum wage in the group, $\max(w_i)g$. As agent i evaluates groups to potentially join, she will find that her counterpart in the group, agent j, will have (1) a much higher wage in lower sacrifice groups, and thus free-ride too much himself, (2) a much lower wage in the high sacrifice groups, and thus will bring too little productive capacity to the table, or (c) have a wage that is "just right," contributing more to the group than she, agent i, will. Put differently, R_j will be increasing with small amounts of sacrifice, as j's comparative advantage in producing R dominates his declining total productive capacity. Eventually, however, the returns to R_j

[14] McBride (2007), offers alternative analysis from the point of view of the group, who themselves stand to benefit from free-riders who can build social capital in the group and contribute more in the future. The group he is modeling, the Church of Latter Day Saints, is a great example of a relatively high, but still moderate, sacrifice group that continues to thrive and grow.

[15]

$$R^e(\pi_S, \pi_R, \beta, I, \alpha) = \left((\alpha^{1/\beta})^{\beta/1-\beta}\pi_R^{1/\beta-1} I\right) / \left(\pi_S^{\beta/\beta-1} + (\pi_R/\alpha^{1/\beta})^{\beta/\beta-1}\right)$$

$$S^e(\pi_S, \pi_R, \beta, I, \alpha) = \left(\pi_S^{1/\beta-1} I\right) / \left(\pi_S^{\beta/\beta-1} + (\pi_R/\alpha^{1/\beta})^{\beta/\beta-1}\right).$$

[16] Specifying the parameters of wage equation with the results of the agent based simulation in this manner is in the same spirit of the method of undetermined coefficients used in differential equations and dynamic stochastic general equilibrium modeling.

[17] We are playing a bit fast and loose with the earlier Nash equilibrium assumption made by agents. The simple Marshallian demand functions are derivable because the agent is assuming the other agent is identical to herself. In setting up $w_j(w_i, A)$, we are setting up an interaction between two agents who are explicitly not identical. Agents in this exercise have demand functions that are derivable because they assume the other agent is identical, even though he is not. Agent rationality is thus heavily bounded, in that they are making decisions using themselves as the model of behavior expected in others, even though their model is false.

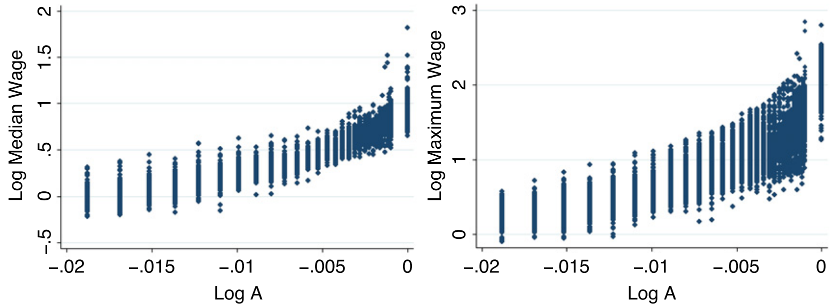

Fig. 8. Log (base 10) group median and maximum wages by log (base 10) A_g. $\beta = 0.7$; $N = 10,000$ in each simulation.

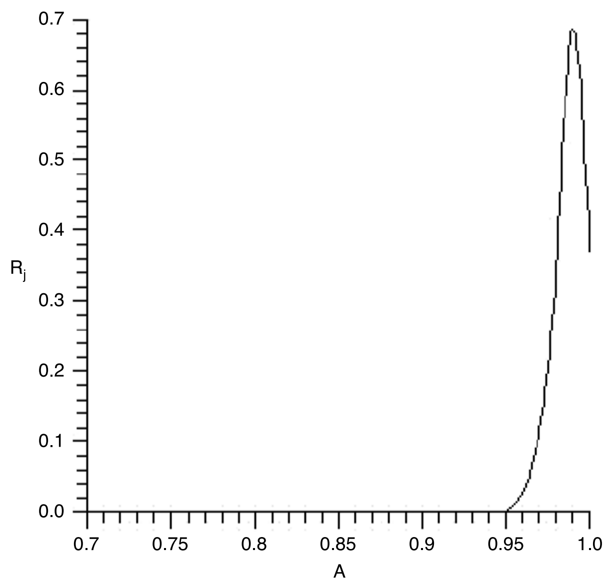

Fig. 9. Production of R by agent i relative to the club's productivity parameter A. Lower values of A correspond to larger sacrifice levels required by the group. $\beta = 0.7$; $w_i = 4$.

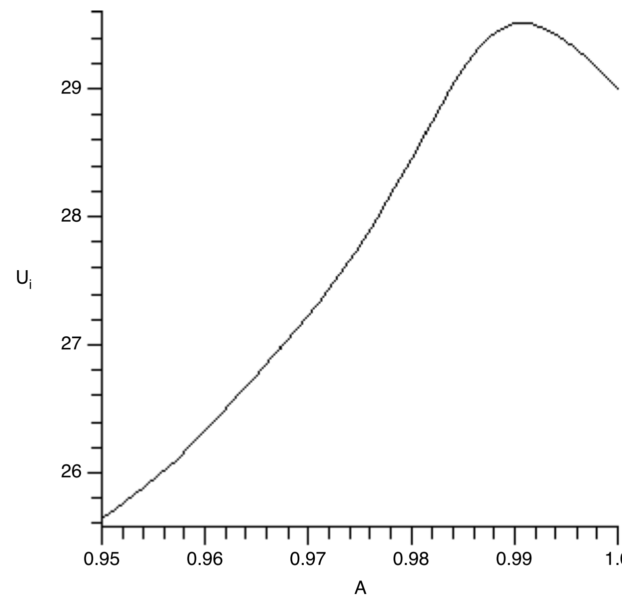

Fig. 10. Utility for agent i relative to the club's productivity parameter A. Lower values of A correspond to larger sacrifice levels required by the group. $\beta = 0.7$; $w_i = 4$.

from sacrifice diminish as agent j's comparative advantage is outweighed by his absolute productive disadvantage derivative of his lower wage. The partner agents from higher sacrifice groups are actually producing a lower quantity of R_j despite dedicating a higher percentage of their full income. This takes the form of an observable local maximum of R_j at a value of A_g that is less than one in Fig. 9.

Derivative of the previous result U_i is increasing with additional sacrifice early on, as the benefits from increasing group efforts of agent j outweigh utility lost to the sacrifice of secular productive capacity. Small sacrifices can attract a membership with a greater commitment to the club's good, in spite of a reduced aggregate productive capacity. This takes the form of an observable local maximum of U_i at a value of A_g that is less than one, but far greater than zero, in Fig. 10.

Sacrifice requirements stand as incentive for members to increase their production of R relative to S, and for high wage agents who are likely to be too great of a free-riding burden to find their membership elsewhere. The sacrifice level serves as a one-way barrier keeping out higher income free-riders. No barrier is needed to keep out lower income agents; they are always desired so long as negative returns to group scale are absent. The emergent group takes shape as higher wage agents find the sacrifice level prohibitive and lower wage agents find the group's religious production insufficient. Within the interval bounded by these thresholds of income, a group finds its membership.

6. Conclusions

Models of signaling, filtering, sorting, and separating equilibria typically reflect an effort by groups to retain within-group agent homogeneity. This is a strategically reasonable assumption on behalf of groups seeking to maximize member utility. It also serves as a strong mechanism for retaining analytical tractability in the models. With these models of perfect sorting and within-group homogeneity, however, it is difficult to explain the success of liberal churches and similarly moderate groups. When the model is allowed deeper agent heterogeneity, such that within-group homogeneity is no longer possible, we can see the merits of moderate sacrifice requirements. Moderate sacrifice can be utility maximizing in such an environment specifically because it serves as a weak screening device for groups, setting an upper bound on the wages of agents that choose to join, while still incentivizing the substitution of group production for private production at the margin. The agents that choose to join groups with moderate sacrifice requirements benefit from the differing comparative advantages of their heterogeneous fellow members.

Moderate sacrifice requirements can be viewed more generally as a means of coping with the weakened altruism associated with diversity within groups (Hungerman, 2009). The interacting mechanics of comparative and absolute advantage that allow moderate religion to persist emphasize the importance of not just mean wages, but the shape of the wage distribution. Within the model, wage inequality has the positive externality of moderating groups, providing an incentive to reduce the demands of sacrifice in an effort to make the group more attractive to prospective members with greater financial means. This incentive to lure wealthier members, in light of evidence regarding intra-congregation skew in the distribution of member donations (Iannaccone, 1997a), has potential implications for the observed decline in strictness and commitment within many denominations that leads, possibly cyclically, to sectarian movements and schism (Montgomery, 1996).

Acknowledgements

I would like to thank Robert Axtell, Laurence Iannaccone, Jared Rubin, Richard Wagner, and an anonymous referee for helpful comments. I would also like to thank the Center for the Economic Study of Religion at Chapman University for their generous funding and support.

Appendix. Imputation of religious commitment variables from the baylor 2005 survey

(1) Income (per year), using categorical responses to Question 60.

(Q60) By your best estimate, what was your total household income last year, before taxes?

$$\begin{aligned}
&\text{Income} = 5000 \text{ if Question } 60 = 1\\
&\text{Income} = 15\,000 \text{ if Question } 60 = 2\\
&\text{Income} = 27\,500 \text{ if Question } 60 = 3\\
&\text{Income} = 42\,500 \text{ if Question } 60 = 4\\
&\text{Income} = 75\,000 \text{ if Question } 60 = 5\\
&\text{Income} = 125\,000 \text{ if Question } 60 = 6\\
&\text{Income} = 200\,000 \text{ if Question } 60 = 7
\end{aligned}$$

(2) Wage (per hour).

$$\text{Wage} = \frac{\text{Income}}{\text{Hours worked last week} \cdot 52}$$

Where "hours worked last week" were numerical responses to Question 62.

(3) Tithe (per year) using categorical responses to Question 11.

(Q11) During the last year, approximately how much money did you and other family members in your household contribute to your current place of worship?

$$\begin{aligned}
&\text{Tithe} = 0 \text{ if Question } 11 = .\\
&\text{Tithe} = 250 \text{ if Question } 11 = 1\\
&\text{Tithe} = 750 \text{ if Question } 11 = 2\\
&\text{Tithe} = 1500 \text{ if Question } 11 = 3\\
&\text{Tithe} = 2500 \text{ if Question } 11 = 4\\
&\text{Tithe} = 3500 \text{ if Question } 11 = 5\\
&\text{Tithe} = 4500 \text{ if Question } 11 = 6\\
&\text{Tithe} = 5500 \text{ if Question } 11 = 7\\
&\text{Tithe} = 6500 \text{ if Question } 11 = 8\\
&\text{Tithe} = 7500 \text{ if Question } 11 = 9\\
&\text{Tithe} = 8500 \text{ if Question } 11 = 10\\
&\text{Tithe} = 9500 \text{ if Question } 11 = 11\\
&\text{Tithe} = 15\,000 \text{ if Question } 11 = 12
\end{aligned}$$

(4) Service Time (translated to hours per year) using categorical responses to Question 5.

(Q5) How often do you attend religious services?

$$\begin{aligned}
&\text{Service time} = 0 \text{ if Question } 5 = 1\\
&\text{Service time} = 1 \text{ if Question } 5 = 2\\
&\text{Service time} = 2 \text{ if Question } 5 = 3\\
&\text{Service time} = 6 \text{ if Question } 5 = 4\\
&\text{Service time} = 12 \text{ if Question } 5 = 5\\
&\text{Service time} = 30 \text{ if Question } 5 = 6\\
&\text{Service time} = 44 \text{ if Question } 5 = 7\\
&\text{Service time} = 52 \text{ if Question } 5 = 8\\
&\text{Service time} = 104 \text{ if Question } 5 = 9
\end{aligned}$$

(5) Religious Activities Time (Monthly).

(Q14(a)–(j)) How often did you participate in the following religious activities last month?

Question 14 a through *j*, religious activities (per month), where "religious time" is the sum of responses to 14 a through *j*.

$x \in \{a, j\}$

$$\begin{aligned}
&\text{Activity Time a} = 0 \text{ if Question } 14\text{x} = 1\\
&\text{Activity Time a} = 3 \text{ if Question } 14\text{x} = 2\\
&\text{Activity Time a} = 7 \text{ if Question } 14\text{x} = 3\\
&\text{Activity Time a} = 15 \text{ if Question } 14\text{x} = 4
\end{aligned}$$

(6) Volunteering through the Church (translated to per Year).

(Q48a) On Average, how many hours per Month do you volunteer through the church?

$$\begin{aligned}
&\text{Volunteering1} = 0 \text{ if Question } 48\text{a} = 1\\
&\text{Volunteering1} = 18 \text{ if Question } 48\text{a} = 2\\
&\text{Volunteering } 1 = 42 \text{ if Question } 48\text{a} = 3\\
&\text{Volunteering } 1 = 90 \text{ if Question } 48\text{a} = 4\\
&\text{Volunteering } 1 = 180 \text{ if Question } 48\text{a} = 5
\end{aligned}$$

(7) Volunteering for the Church (translated to per Year).

(Q48c) On Average, how many hours per Month do you volunteer for the church?

$$\begin{aligned}
&\text{Volunteering2} = 0 \text{ if Question } 48\text{c} = 1\\
&\text{Volunteering } 2 = 18 \text{ if Question } 48\text{c} = 2\\
&\text{Volunteering } 2 = 42 \text{ if Question } 48\text{c} = 3\\
&\text{Volunteering } 2 = 90 \text{ if Question } 48\text{c} = 4\\
&\text{Volunteering2} = 180 \text{ if Question } 48\text{c} = 5
\end{aligned}$$

(8) Total Volunteering = Volunteering1 + Volunteering2.

These imputed factors allow for the following calculation:

(9) Religious time = Total Volunteering + (Activity time · 12) + Service time.

(10) Religious fraction $= \frac{(\text{Religious time} \cdot \text{Wage}) + \text{Tithe}}{\text{Full Income}}$.

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